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THE EARTH'S FREE OSCILLATIONS

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SUMMARY

The Chilean earthquake of May 1960 provided geophysicists with the most unique data yet obtained regarding the interior of the earth. Detailed analyses of this data have conclusively demonstrated that the earthquake excited the free vibrations of the earth. Theoretical predictions of the resonant modes of the earth and subsequent observations have substantiated and refined the physical model of the earth. A modified Gutenberg model of the mantle has yielded a comparison between experiment and theory to within better than 1 percent.

G-194

CONTENTS

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INTRODUCTION

The great Chilean earthquake of May 22, 1960, excited the earth's free modes of vibration and these were observed for the first time. The detection of the free modes broadens the spectrum over which a geophysicist may look into the earth's interior. Prior to 1960, almost all information regarding the earth's interior had been derived from detailed investigations of the arrival times of elastic body waves as recorded by seismographs. These waves travel different paths through the earth and contain most of their energy in the high frequency part of the spectrum (0.1 to 10 cps). The interpretation of their arrival times is based on a ray theory similar to the ray theory of geometrical optics. Thus, use of low frequency normal modes as a tool for the investigation of the earth is somewhat analogous to the astronomer s use of radio frequencies as a supplement to observations in the visual range.

FREE VIBRATIONS OF AN ELASTIC SPHERE

The observation and interpretation of the free oscillations in the latest chapter in an investigation begun in 1882 by the noted mathematical physicist Horace Lamb. If an elastic solid is tapped by a hammer, the elastic disturbance is initially carried outward by two traveling waves. The fast wave (P wave) carries with it the compression and rarefaction of ordinary sound. The slow wave (S or shear wave) transmits particle motion at right angles to the direction of propagation. If the solid is sufficiently isolated from its surroundings, reflections from boundaries may set up standing waves; the solid then rings or vibrates at the normal mode frequencies.

In 1880, Jaerisch (Reference 1) carried out an investigation of the toroidal and radial vibrations of a homogeneous sphere. In 1882, Lamb (Reference 2) showed that the free vibrations of an elastic sphere can be classified into two groups: the *toroidal* or *torsional* oscillations and the *spheroidal* oscillations. The toroidal oscillations are

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those in which a particle executes motion on a spherical surface; there is no radial component of motion. The toroidal oscillations unite to form the familiar horizontally polarized sheer waves (SH waves) of classical seismology. The spheroidal oscillations combine both radial and tangential motion to produce compression and rarefaction. A degenerate spheroidal oscillation involves only radial motion; the entire sphere expands and contracts.

The notation adopted to describe the earth's free oscillations is similar in many ways to the notation used in atomic spectroscopy, thus reflecting the common mathematical structure of these two fields. The solution to the equations of motion of an elastic sphere can be separated into a function dependent upon radius and a function dependent on angular coordinates. The angular function is written as a sum of surface spherical harmonics:

$$X_l^m = P_l^m (\sin\theta)\, e^{im\phi}, \tag{1}$$

where P_l^m is the associated Legendre function. If a geographical coordinate system is used, then ϕ is the longitude and θ is the latitude. The time dependence can be included in the exponential as $\exp[i(m_\phi - \omega t)]$, where ω is the angular frequency. This represents a wave traveling with a speed of (m/ω) radians/sec. If m is positive, the wave travels from west to east; if m is negative the wave travels from east to west. In a stationary sphere, the two signs of m are symmetrical. Rotation, however, destroys the symmetry and creates important differences between waves traveling from west to east and waves traveling from east to west.

The numbers m and l are familiar in quantum mechanics as the magnetic and azimuthal quantum numbers respectively. They must assume integral values, and these integers determine the surface pattern of deformation associated with a particular free oscillation. The number of lines of vanishing displacement associated with the angular coordinate θ is $(l - |m|)$; the number of nodal lines associated with the angular coordinate ϕ is m. There will also be surfaces of zero particle displacement associated with the radial function. Free oscillations can thus be characterized by three integers: l and m determine the pattern of displacement on the spherical surface and n determines the number of internal nodal surfaces. The notation that has been adopted is ${}_nS_l^m$ and ${}_nT_l^m$ for spheroidal and toroidal oscillation, respectively. The notation ${}_nT_l^m$ denotes a toroidal oscillation with n radial nodal surfaces and a displacement pattern on the surface of the sphere fixed by the surface spherical harmonic with ordinal numbers l and m. In the ${}_0S_2$ oscillation, a sphere alternately assumes a prolate and oblate form; it is sometimes termed the football mode. In the ${}_0T_2$ oscillation, one hemisphere rotates differentially or twists relative to the other.

Lamb (Reference 2) treated a homogeneous uniform sphere. It was recognized by Jeans (Reference 3) that gravity influences the spheroidal oscillations since these involve radial motion. At low frequencies, the gravitational forces are of the same order of magnitude as those produced by elastic distortion; indeed it can be shown that over large regions of the earth there is a delicate balance between the two forces. A method of treating gravitational effects was suggested by Rayleigh (Reference 4) and later employed by Love (Reference 5) and by Jeans (Reference 3). Rayleigh's method involves considering the strain as measured from the initially hydrostatically stressed state.

The solution to the problem of gravity failed to remove all difficulties in computing the periods of the free oscillations. The earth is inhomogeneous, its density and elastic properties varying in a radial direction. It is now known that the inner half of the earth contains a liquid core with a small inmost core that is thought to be solid.

The reduction of observations of numerous earthquakes at many stations leads to estimates of the variations of the P and S wave velocities. These velocities can be combined with the earth's moment of inertia and mass to obtain an estimate of the density variation within the earth. (A clear and detailed discussion is given in Reference 6.) One such model (Figure 1) of the earth's interior is due to Gutenberg (Reference 7); also see Reference 8. The characteristic feature of the Gutenberg model is a dip in the seismic velocities beginning at the crust-mantle boundary and extending to a depth of 150 kilometers. Alternative models have been constructed by Bullen (Reference 6) and Jeffreys. In these models the seismic velocities monotonically increase with depth; there is no region of low velocity (Figure 2). A particular model suggested by Bullen, Bullen Model B, contains an inner core of much higher density (17.9 gm/cm^3) than that postulated in the Gutenberg model.

The extension of the theory for a homogeneous sphere to an inhomogeneous earth involves formidable computational problems. A variational method has been suggested by Stoneley (Reference 9) and much of the work since that date has been based on this approach. Recent applications of the variational method have been made by Jobert (Reference 10) and Takeuchi (Reference 11). Pekeris and his coworkers proposed direct numerical methods which made possible a massive computational effort; electronic computing machines were used. These numerical techniques are described in Reference 12. Alternative numerical schemes are discussed in Reference 13, and in Reference 14 where it is suggested that the spectral lines would be split by the earth's departure from sphericity and by its rotation.

The effects of the inhomogeneous elasticity and density on the free oscillations are illustrated in Figures 3 and 4 where the Gutenberg model earth is contrasted with a homogeneous model in which the elasticity equals the average elasticity of the Gutenberg model. The energy levels shown in Figure 3 are normalized to provide a 1 centimeter displacement at the surface. About 3×10^{21} ergs are required to produce this displacement with a ${}_0T_2$ surface pattern in both the homogeneous and inhomogeneous models. At higher modes and higher frequencies (Figure 4) it takes more energy to form the more complicated surface pattern of displacement while maintaining a maximum surface

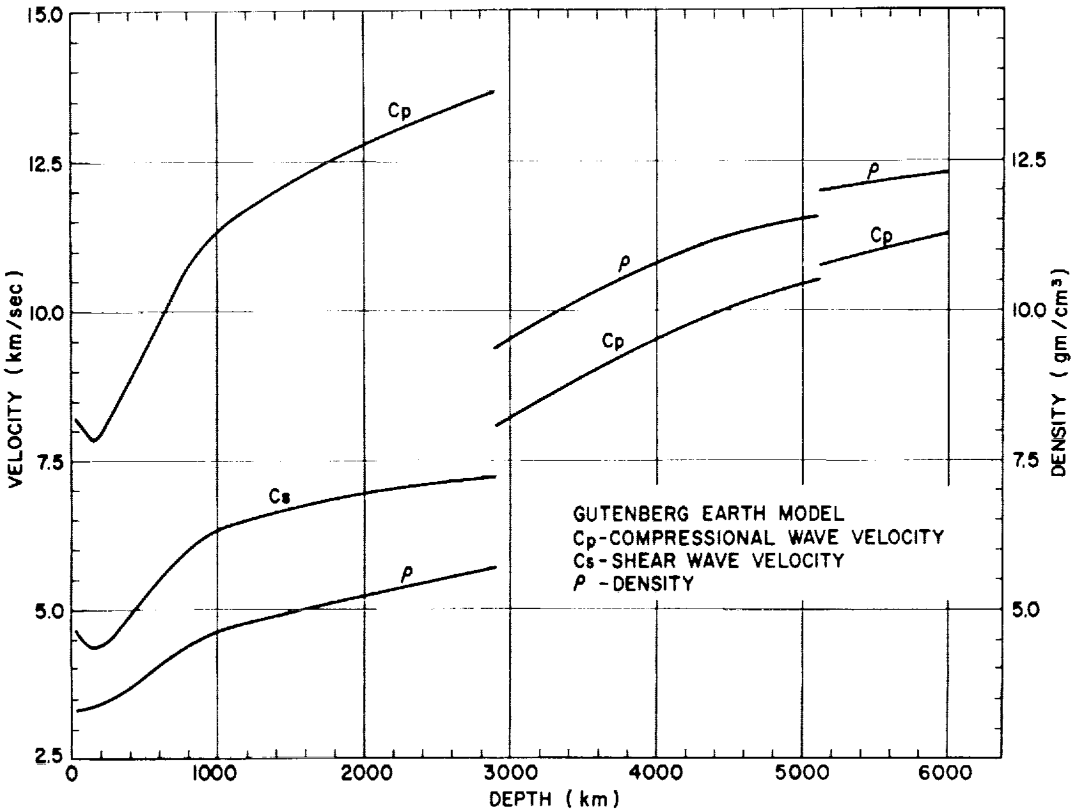


Figure 1—Variation of the compressional and shear-wave velocity and density within the earth according to Gutenberg (Reference 7)

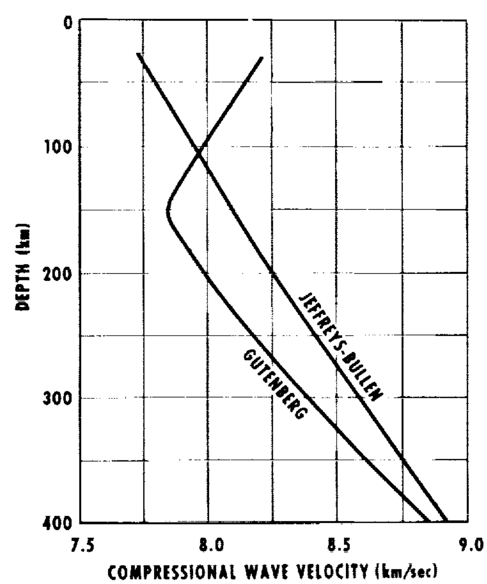
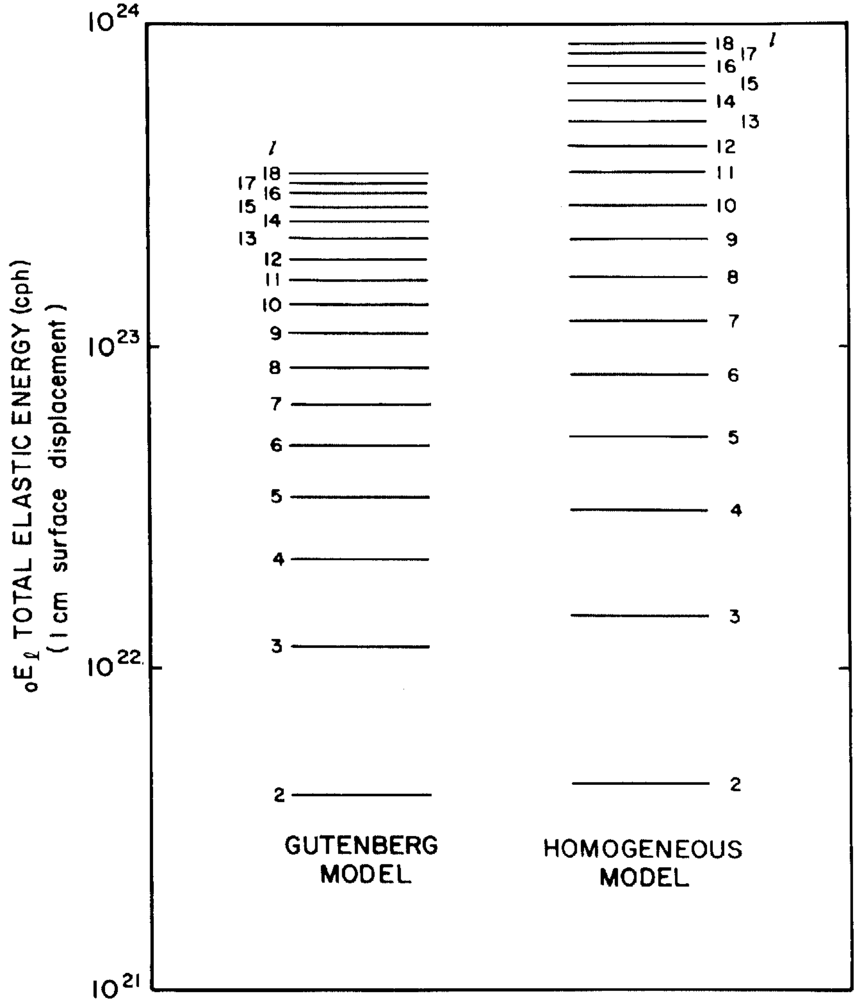
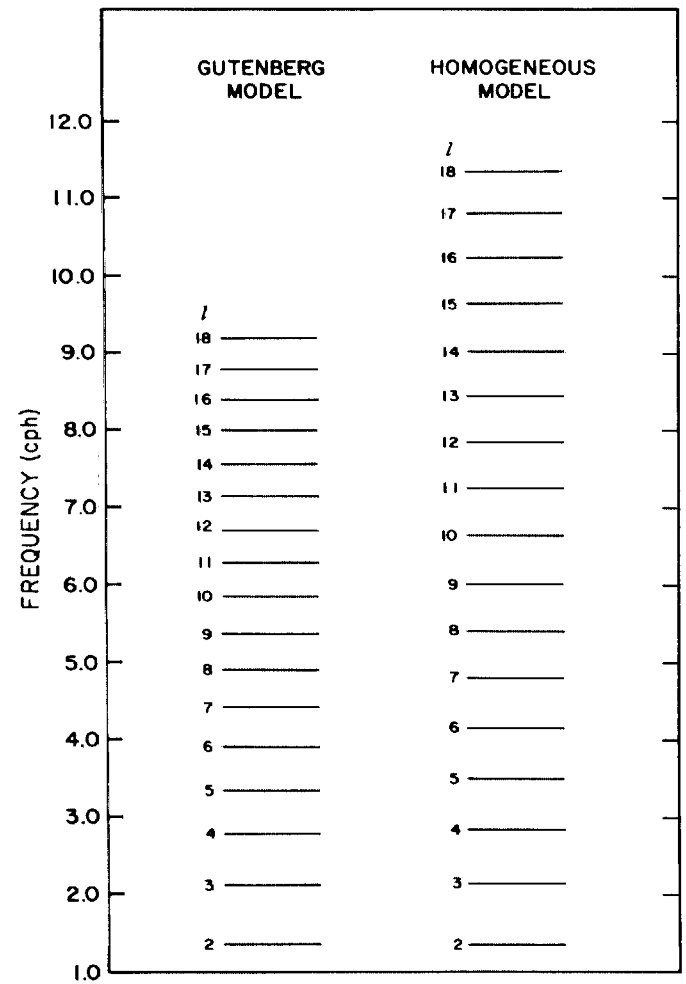


Figure 2—Comparison of the Gutenberg and Bullen models in the upper mantle



ENERGY LEVELS FOR $_0T_l$

Figure 3—Total elastic energy in the fundamental toroidal oscillation. The energy is normalized to a 1 cm surface displacement (after MacDonald and Ness, Reference 13)



FREQUENCIES FOR $_0T_l$

Figure 4—Comparison of resonant frequencies for the toroidal oscillations in Gutenberg and homogeneous model earth (after MacDonald and Ness, Reference 13)

amplitude of 1 centimeter. Moreover, the needed energy is greater in the homogeneous model, since the near-surface rigidity is larger than in the Gutenberg model; and this difference increases with increasing values of the mode number l. At low mode numbers the energy is more or less evenly distributed over the entire mantle (Figure 5), but at higher mode numbers the elastic energy is concentrated in the outer layers of the mantle. Thus the ${}_0T_2$ oscillation involves the mantle; the ${}_0T_{18}$ oscillation is confined to the upper few hundred kilometers.

A further development in the theory should be noted. Jeans (Reference 3) showed that the free oscillations excited by an earthquake could be regarded as a system of dispersive surface waves and other waves diffusing into the earth's interior. He established the correspondence of normal mode theory with ray theory in an elegant way. The ray-wave theory emphasizes the high frequency part of the spectrum, the normal mode theory the low frequencies.

OBSERVATION OF THE EARTH'S FREE OSCILLATIONS

Despite the considerable theoretical efforts, only recently was an attempt made to observe the earth's free oscillations. Benioff (Reference 15) constructed a strain

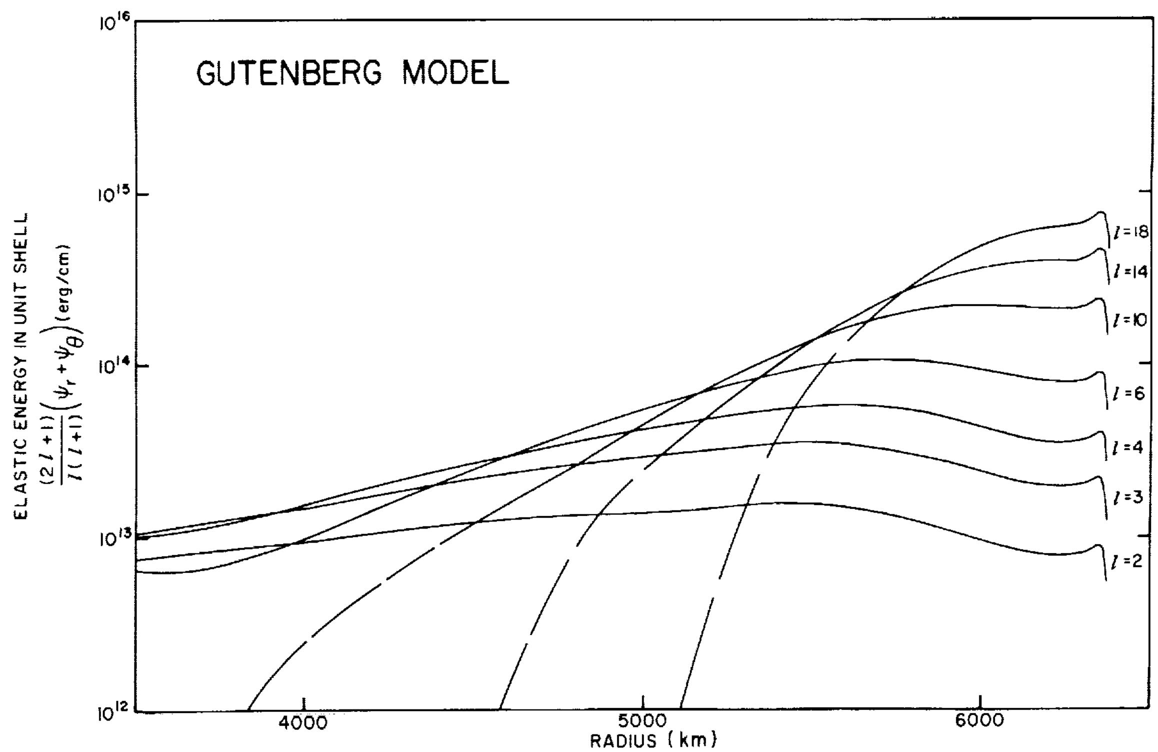


Figure 5—Total elastic energy per unit radius in the toroidal oscillations for the Gutenberg model earth (after MacDonald and Ness, Reference 13)

measuring seismometer, in the form of a silica glass rod 24 meters long, with the particular purpose of investigating the low frequency spectrum. He suggested that an apparent 57 minute periodicity visible on the strain records of the Kamchatka earthquake of 1952 was the ${}_0S_2$ mode, and this single suggestion prompted extensive computational work. A second attempt at detecting the free oscillations was made in 1958 by spectrally analyzing the background noise in the strain seismometer and in the changes of the local gravitational field (Reference 15). Finally, between 1958 and 1960 several instrumental developments made possible the observation of the free oscillations excited by the Chilean earthquake. Benioff, at the Seismological Laboratory of the California Institute of Technology, modified the circutry associated with the strain seismometer so that the effects of the finite-amplitude earth tides were reduced and a greater magnification was permitted. Also, a lower noise level was achieved on the LaCoste-Romberg gravimeter operated by the Institute of Geophysics at the University of California, Los Angeles. In addition, the Lamont Geological Observatory installed a strain gauge of the Benioff type in a mine shaft near Ogdensburg, New Jersey.

The free oscillations by the Chilean earthquake were detected on both the gravimeter and the strain seismometers. These instruments complement each other: The strain seismometer is sensitive to strain produced both by vertical and horizontal motion; it therefore records both spheroidal and toroidal oscillations. On the other hand, the gravimeter records only vertical accelerations and spheroidal oscillations. A combination of the observations from both instruments allows the separation and identification of the two classes of motion.

The power spectrum of the variations in gravity at Los Angeles for the four days following the Chilean earthquake is shown in Figure 6. Figure 6 should be compared with Figure 7, a record of a quiet interval 116 hours in duration a month after the earthquake. This spectrum is almost structureless, though there is a peak at 20.5 minutes. The spectrum of a seismic disturbance is thus characterized by well defined sharp peaks for periods between an hour and about eight minutes. At higher frequencies the isolated peaks begin to merge into a continuum as a result of the finite width of an individual peak and the increased number of peaks. Similar observations and analysis of the resulting data were carried out on records of strain by Benioff, Press and Smith (Reference 16) and Alsop, Sutton, and Ewing (Reference 17). In 1960, Press reported to the American Geophysical Union the presence of higher order free oscillations in seismic records of large earthquakes.

DETERMINATION OF EARTH MODELS

A comparison between the spheroidal modes calculated by Pekeris, Alterman, and Jarosch (Reference 18) and those measured by Ness, Harrison, and Slichter (Reference 19) is shown in Figure 8. At low frequencies the observations favor neither model, but at higher frequencies the observations closely fit the Gutenberg model.

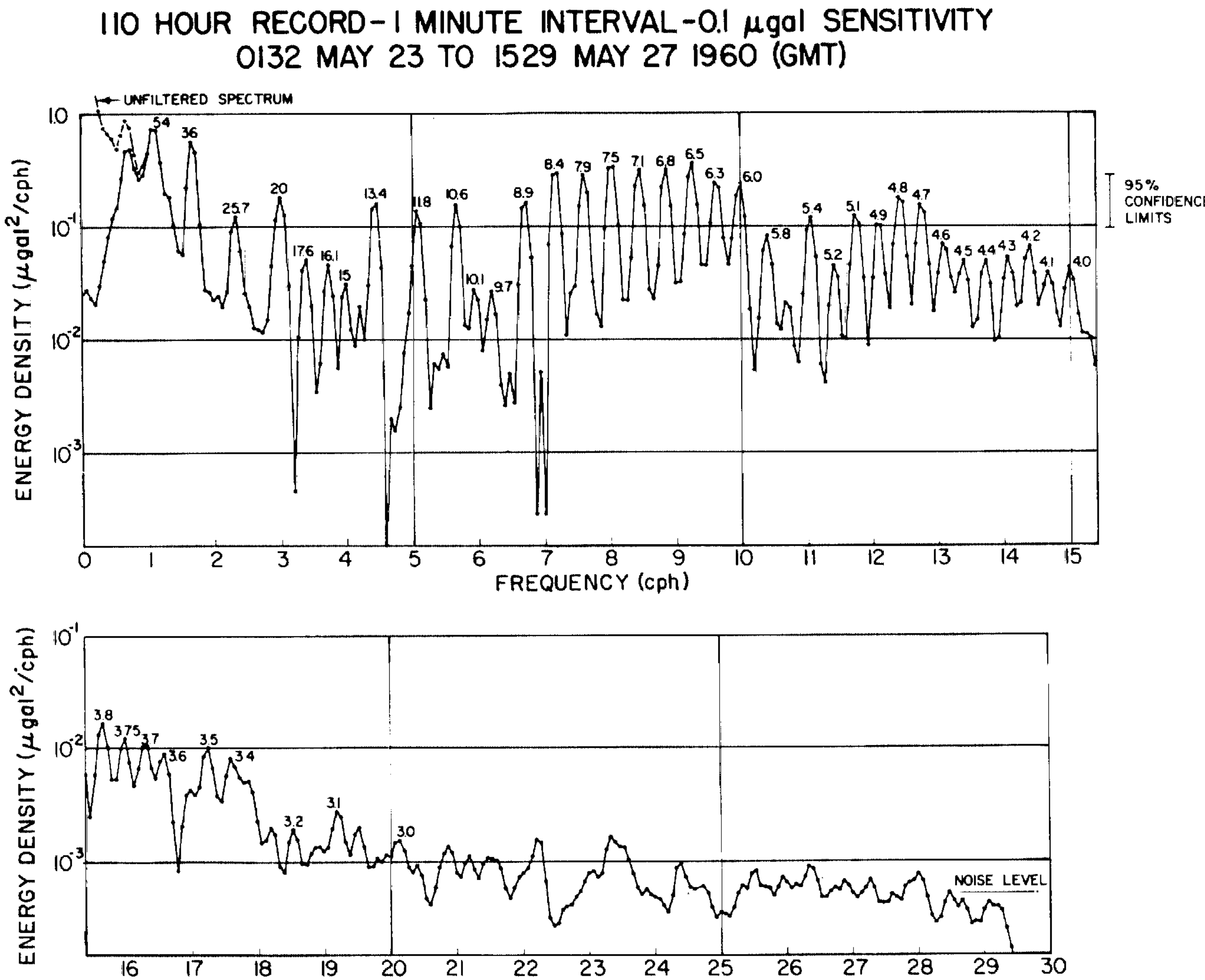


Figure 6 — Power spectrum of a gravity record taken of UCLA after the Chilean earthquake of May 22, 1960, (after Ness, Harrison, and Slichter, reference 19)

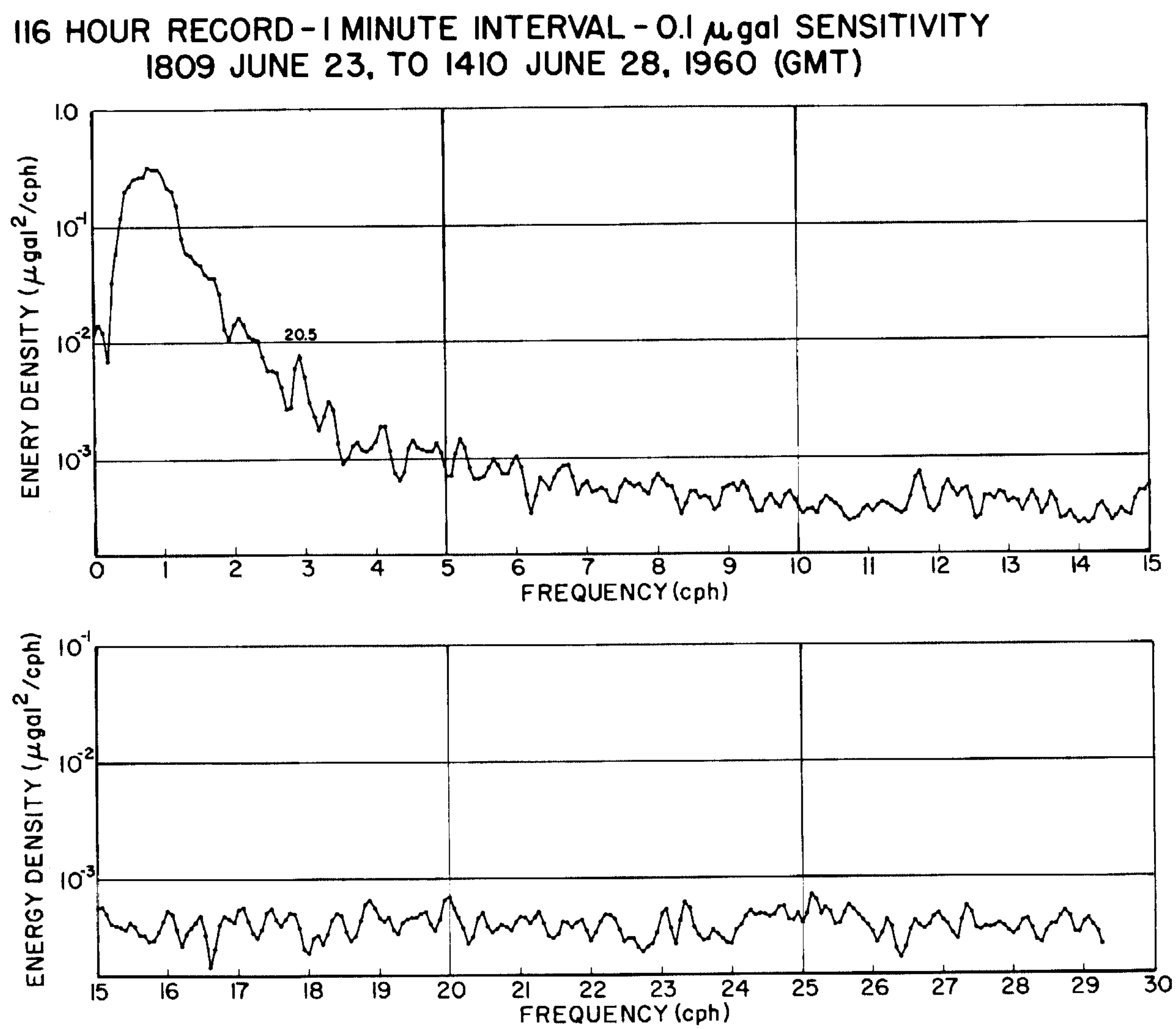


Figure 7 — Power spectrum of a quiet period recorded at UCLA a month after the Chilean earthquake of May 22, 1960, (after Ness, Harrison, and Slichter, reference 19)

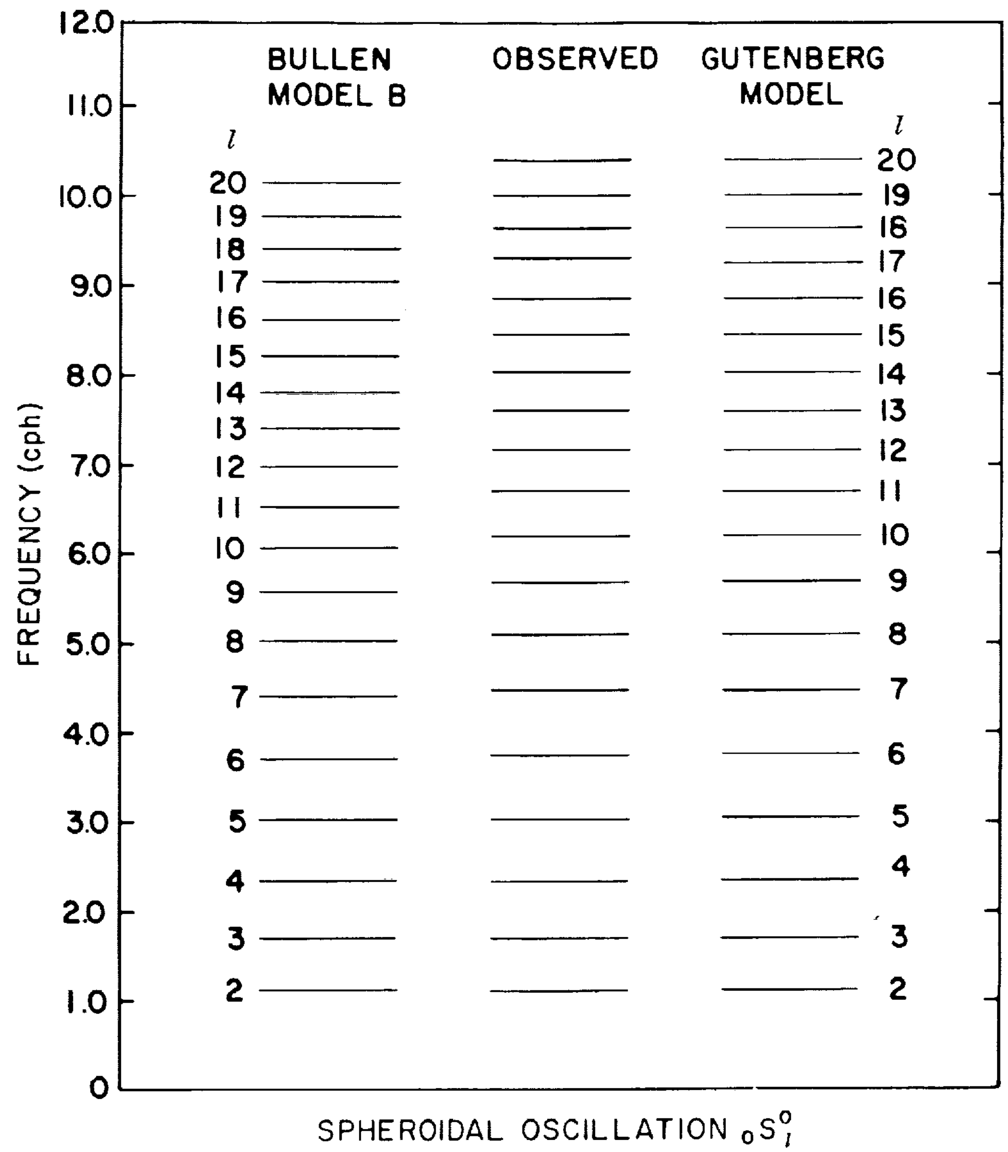


Figure 8—Comparison of calculated and observed frequencies of the spheroidal oscillations

Table 1 lists a comparison between the calculated and observed toroidal oscillations. In addition to the Gutenberg model shown in Figure 1, a modified Gutenberg model, Gutenberg IV, is included. The observed periods are consistently shorter than those of the Gutenberg model in the low-order oscillations and the periods are more nearly equal in the high-order oscillations. This suggests that, on the average, the Gutenberg model has too high a rigidity. The perturbation of the Gutenberg model to form Gutenberg IV is controlled by the fact that the low-order oscillations involve the entire mantle of the earth while the higher-order oscillations reflect the properties of only the outer few hundred

Table 1

Comparison of Calculated and Observed Periods of Toroidal Oscillations (in Minutes)

Order	Calculated Period			Observed Period
	Bullen B Model	Gutenberg Model	Gutenberg IV Model	
2	44.18	43.63	44.11	42.94
3	28.62	28.25	28.55	28.57
4	21.92	21.64	21.86	21.95
5	18.09	17.86	18.04	18.02
6	15.55	15.37	15.52	15.51
7	13.72	13.60	13.72	13.75
8	12.33	12.24	12.35	12.35
9	11.23	11.17	11.26	11.24
10	10.35	10.29	10.38	10.33
11	9.59	9.56	9.64	9.614
12	8.95	8.94	9.01	9.065
14	7.92	7.93	7.99	7.985

kilometers. The Gutenberg IV model is produced by reducing the shear-wave velocity throughout the lower part of the mantle by 2 percent and maintaining the Gutenberg velocity distribution in the outer 400 kilometers.

The comparisons of the toroidal and spheroidal oscillations with models of the earth strongly indicate a preference for a model with a region of low velocity. This provides additional supporting evidence for the existence of a region of low velocity (Reference 20).

In the Gutenberg model, the velocity increases everywhere in the mantle but in a thin near-surface region. The conditions which will give rise to the anomolous decreasing velocity are of great interest. Laboratory measurements show that in silicates the wave velocity increases with increasing pressure; pressure stiffens a rock. An increase in temperature has the opposite effect, decreasing the wave velocity. In the outer regions of the earth, both pressure and temperature increase; thus the velocity will decrease if the increase of temperature wins out over the increase of pressure. An extrapolation of laboratory data indicates that a temperature gradient of 6 to 7 deg/km is sufficient to produce a decrease in velocity (References 13 and 21). It is then of interest to inquire as to what distributions of radioactive heat sources and thermal conductivity are sufficient to give the required critical temperature gradient and at the same time account for the heat flowing from the earth's interior.

It is generally assumed that the radioactive heat sources are concentrated towards the surface and the concentration is greater under continents than under oceans. The thermal conductivity may vary because of the contribution of radiation at high temperatures. This combination of the near-surface concentration of heat sources and a thermal conductivity increasing with depth requires that the steepest temperature gradient exist in the upper mantle. If the low velocity zone is indeed due to a high temperature gradient, then the low velocity zone should begin at the base of the crust rather than at some greater depth, 100 to 200 km, as has been suggested on the basis of studies of near earthquakes.

The thermal conditions that could give rise to the low velocity zone are illustrated in Figures 9 and 10. Conditions approximating the upper mantle under oceans are shown in Figure 9. The radioactivity is concentrated in the upper 430 km and there is no further concentration of radioactivity near the surface. The calculated temperature gradient exceeds the gradient required to produce a low velocity zone at depths ranging from 100 to 150 km. Figure 10 illustrates the conditions that might be expected under continents. Here radioactivity is distributed over the upper 430 km but half of it is placed above 30 km. This near-surface concentration of radioactivity reduces the temperature gradient, and a low velocity zone exists at depths down to 50-100 km. The calculations suggest that there should be a marked difference between the extent of the low velocity zone under continents and its extent under oceans. It should extend to greater depths and be better developed under oceanic areas.

The low velocity zone might be due to large scale chemical inhomogeniety in the upper mantle. Under that hypothesis, it could be found at greater depths. Detailed studies of surface waves and the use of artificial explosions are needed to understand the origin of the low velocity layer.

EARTHQUAKE ENERGY

Estimates of the energy released during a large earthquake differ by factors of ten. A source of uncertainty is the energy contained in the low frequency end of the spectrum. The energy levels of the toroidal oscillation shown in Figures 3 and 4 can be combined with observations on particle displacement at the surface to yield an estimate of the energy in the earthquake at these low frequencies. The displacements obtained by the strain seismometers in the Chilean earthquake give an energy density in the ${}_0T_2$ mode of 5×10^{18} ergs/cph, and the total energy in the toroidal oscillations having periods greater than 9 minutes is about 10^{20} ergs. If there is equipartition of energy between the toroidal and spheroidal oscillations, then about 10^{21} to 10^{22} ergs of energy were initially present in oscillations with periods greater than 1 minute. These figures should be compared with the estimated 10^{24} ergs total elastic energy released by the Chilean earthquake.

G-194

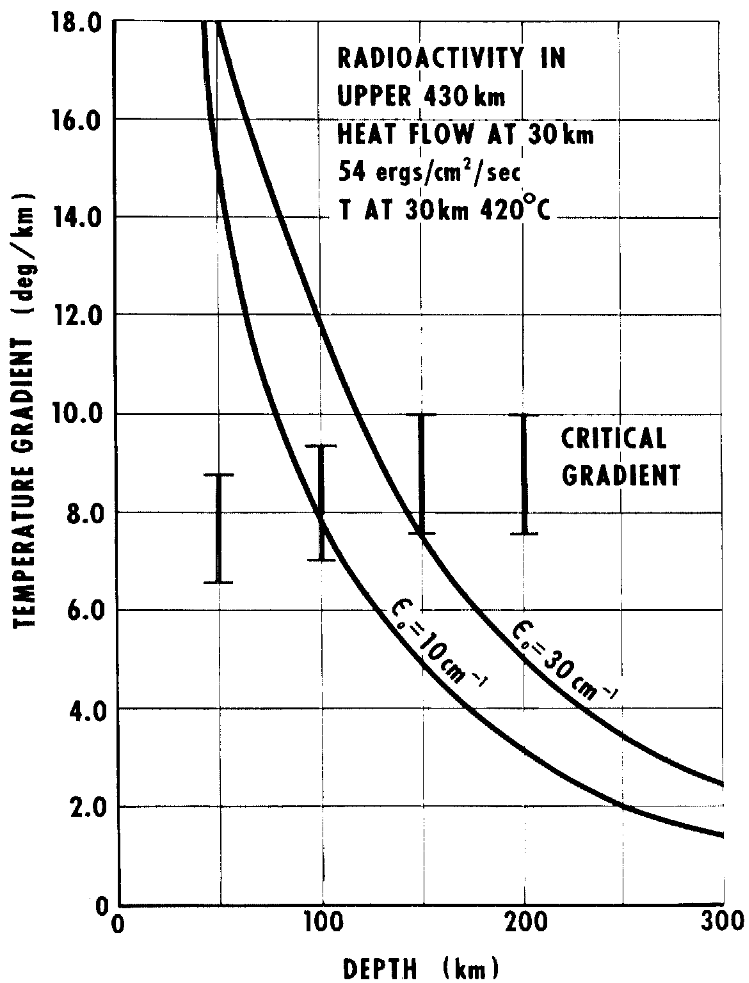


Figure 9—Computed thermal gradient under a typical ocean. The critical temperature gradient required to produce a low velocity zone is indicated by the vertical heavy lines.

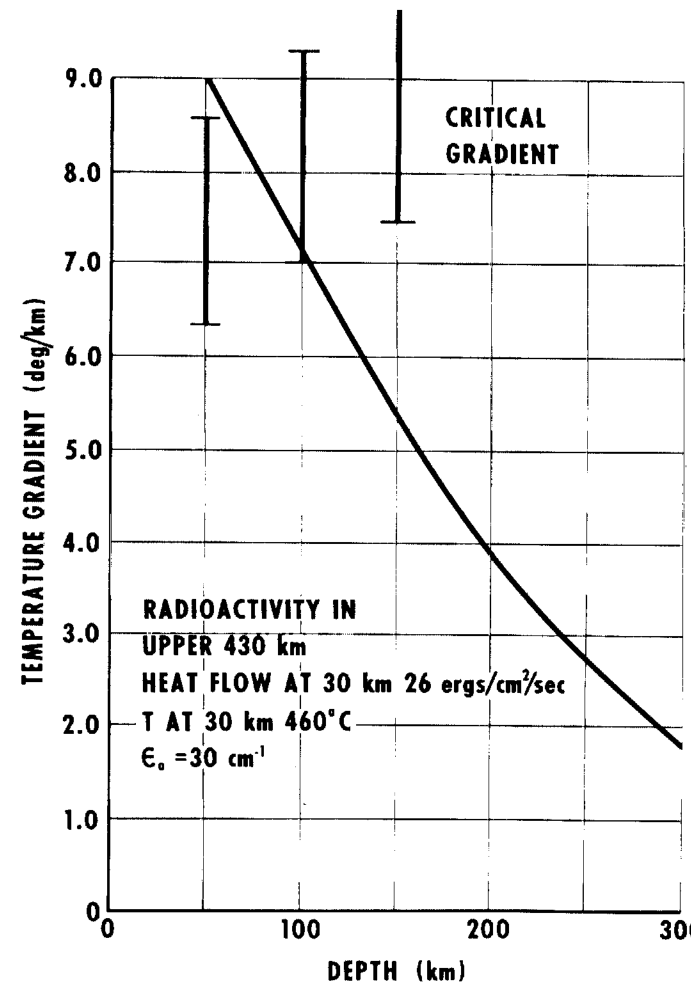


Figure 10—Computed thermal gradient under a typical continental region. The critical temperature gradient is indicated by the heavy vertical lines.

LINE STRUCTURE

Ness (Reference 19) and Smith (Reference 16), in reducing the data from the Chilean earthquake, noted that the low frequency spectral peaks appeared as doublets or triplets instead of single lines as would be expected for a stationary elastic sphere. Rotation destroys the symmetry with respect to the integer m in Equation 1, and the degeneracy associated with the symmetry is removed. The effect of rotation on the oscillations can be qualitatively understood by recalling that a free oscillation is composed of a number of running waves. Those waves traveling in the direction of the earth's rotation are carried forward relative to those waves traveling in the opposite direction. The net effect is that the total pattern of surface deformation rotates relative to the earth. The local effect is to cause the vibrating particles to precess much in the fashion of a Foucault pendulum. The rotational splitting is analogous to the Zeeman effect in spectroscopy, in which a magnetic field removes the degeneracy with respect to the quantum number m.

Detailed calculations of the splitting in spheroidal and toroidal oscillations have been carried out (References 13, 22 and 23). The calculated splitting is in agreement with that observed in the low order spheroidal oscillations; an oscillation of order l being split into $(2l + 1)$ peaks. The fine structure of the lowest order toroidal oscillation is in doubt and the line ${}_0T_2$ presents a number of problems to be taken up shortly. It should be remarked that rotational splitting of the elastic vibrations is analogous to the effect of rotation on the axisymmetric oscillations of a fluid sphere. These oscillations have been studied in detail by astronomers concerned with variable stars. Indeed, Cowling and Newing (Reference 24) obtained, for the rotational frequency shift in the free oscillations of a star, an expression identical in form to that describing the effect of rotation on the elastic vibrations of the earth.

If the earth were a perfectly elastic body, then the spectral peaks should show up as individual lines broadened only by the data reduction techniques (instrumental broadening). The deviations from perfect elasticity or fluidity, however, result in a natural broadening of the lines. The degree to which a given line is broadened or, alternatively, the rate at which a given peak decays in time, provides a measure of the anelastic properties of the earth. The distribution of the anelastic properties can be obtained by studying the decay rate of oscillations of various frequencies since differing frequencies represent different portions of the earth. Furthermore, several mechanisms of dissipation will be prominent in the various oscillations. The spheroidal oscillations of order 2 involve the entire earth, including the core, and the motion contains components of compression and shear. Only compressional motion is involved in the radial oscillation ${}_0S_0$ and this oscillation provides a measure of the dissipation of the earth in compression. The broadening of the toroidal lines is due primarily to dissipative processes within the mantle. An additional sink of energy is provided by the interaction of the core and mantle. A detailed study of possible viscous and hydromagnetic effects rules out the core-mantle boundary as a major contributor to the energy loss (Reference 13).

The half-width of the lines Q (the rate of energy dissipated per cycle divided by the peak elastic energy) is found to be about 350 for spheroidal oscillations. Thus, in spheroidal oscillation the earth rings as a rather poor bell. The estimates of the Q for toroidal oscillations are less good, but somewhat lower figures are indicated. The highest Q of all is shown by the radial oscillation: As is noted in Figures 6 and 7, the earth appears to have been ringing in this mode of oscillation a month after the earthquake. The indicated Q is greater than 1000; thus, the dissipation in compression is much less than in shear. Such a conclusion is in agreement with the suggestion by Knopoff and MacDonald (Reference 25) that the major mechanism for dissipation of small amplitude waves in the earth is frictional rubbing across grain boundaries.

CORE PROBLEMS

Smith (Reference 26), in a careful analysis, finds that the period of the earth's fundamental toroidal oscillation is 42.94 minutes. This period is more than a minute less than the period predicted for the Gutenberg IV model which gives a reasonable fit to the other oscillations. The deviation is in the direction that would be expected if the core-mantle boundary were partly rigid; the resonant period for Gutenberg IV mantle with a rigid inner surface is 32.1 minutes. A possible explanation of the apparent stiffness involves the earth's magnetic field. A component of the magnetic field tangential to the core-mantle boundary leaks out of the core into the conducting lower mantle and combines with the dipole component to give a Maxwell stress. The lower mantle is then partially glued to the core, and this leads to an apparent stiffness. If this interpretation is correct, an estimate can be made of the conductivity of the lower mantle and the strength of the magnetic field. Detailed studies of toroidal oscillations of low frequency may lead to fundamental information regarding the electromagnetic properties of the core and lower mantle.

Slichter (Reference 27) has emphasized the presence of a peak at the low frequency end of the spectrum that is not theoretically predicted (see Figure 6). A possible interpretation of this peak is that it represents the jiggling of the solid inner core, and the jiggling is dampened by interaction with the fluid outer core. The problem remains open however, since the reality of the peak needs to be established and the detailed dynamics of the inner core oscillations need exploration.

CONCLUSION

The development of sophisticated instrumentation has permitted the investigation of the earth's interior in a new frequency range, and the results from a single earthquake have provided abundant new information on this subject. Free oscillations of the earth are excited only by large earthquakes. Over the next few years a few large earthquakes may be expected, and the associated vibrations will yield new limits on the internal constitution of our earth.

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National Aeronautics and Space Administration.
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